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Mathematical practical sessions with manipulatives: Trainee teachers' perceptions of their utility

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This study presents the perceptions of trainee teachers regarding the realisation of practical mathematical sessions to expand on the concepts that are taught in primary education using manipulatives. For a 4-month period, a practical session with the mentioned characteristics was carried out weekly with Year 1 students of the Primary Education Teaching Degree. After their completion of the module the students were requested to comment in writing on the utility of such practical sessions. The resulting information was analysed qualitatively through content analysis using ATLAS.ti software (Version 7.5.4). Six broad categories grouping the utilities considered by the students were found.

Keywords: mathematics; mathematical practical sessions; teacher education; trainee teacher

Introduction

For some decades now research into teachers and their initial training has exposed that the knowledge which teacher trainees have about teaching is influenced by their learning experiences as primary and secondary education pupils (Chuene, Lubben & Newson, 1999; Comeaux, 1991; Sim, 2006). These findings point at the need of trainee teachers, both for primary and secondary education, to have access to new learning experiences that they may later embed in their teaching practice.

In this sense, the recent creation of the European Higher Education Area (EHEA) prompts a shift in university-level instruction towards an education based on the acquisition of competences and centred on students' learning process, which brings about the needs or failures in teacher training programmes.

Authors like Calderhead (1991) have highlighted that trainee teachers must be provided with opportunities for them to acquire different teaching strategies as compared with those they have previously experienced; this will provide them a greater variety of resources, strategies, and methods to use in their future performance as teachers. As for instruction in mathematics, it is particularly important to attend to such needs because students frequently show difficulties in the subject and also because, ultimately, mathematics is formal and abstract in nature.

Consequently, we seek to reveal the opinions of trainee teachers regarding the realisation of practical activities in the classroom in which manipulatives are used to promote a positive and familiar view of mathematics, since such opinions can be useful and applicable in teacher training programmes in other countries.

Theoretical Framework

Internationally, different approaches and strategies have been adopted to improve trainee teachers' learning (Avalos, 2011; Van Hover & Hicks, 2018). These approaches and models vary in content, process and context. For instance, an experience of professional development may revolve around the implementation of a new teaching method or around the exploration of solutions to a problem based on the difficulties faced in the classroom (Goodnough, 2010). In any case, the new approaches to teacher training facilitate the implementation of learning experiences that, grounded on the collaboration between students, allow to link with the daily experience of teachers and with validated good teaching practices (Goodnough, 2010; Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003).

Comparative studies – in Asian and Western countries – that include classroom experiences have demonstrated that teachers' practices across cultures are based on pedagogies that privilege different forms of student action, which, in turn, are grounded in different theories of learning (Xu & Clarke, 2013).

Trainee teachers' learning is a process during which their knowledge and forms of reasoning become increasingly closer to those of more experienced teachers (Maz-Machado, León-Mantero & Renaudo, 2015). This process is characterised by its generation from active student engagement in contexts defined by ordinary cultural practices; where both students and professors use their previous knowledge and give it a new didactic and conceptual meaning (García, Sánchez, Escudero & Llinares, 2006). Therefore, this engagement has an influence on trainee teachers, expanding or modifying the meaning of the conceptual instruments that are used.

All these considerations lead to initial teacher training university programmes to aim, as a priority, for students to begin to develop some of the many competences which are necessary for their future performance as

professional teachers. As stated by Llinares (2009), the goal is to have them acquire specific didactic knowledge they may use in teaching-learning situations.

Practical sessions in mathematics teacher training

A focus of recent research in the field of mathematics education centres on mathematics teachers' own knowledge, conceptions, beliefs, attitudes, competences, practice, and identity (Da Ponte, 2012). This reflects the wide variety of processes and issues that concern and naturally form part of the process of teaching and learning mathematics within compulsory education.

Anthony and Walshaw's (2007) study reveals that the mathematical activities that teachers carry out in their classrooms depend greatly on what they know and believe about mathematics, as well as on their understanding of the processes of teaching and learning. It is a known fact that teachers who are successful in their classrooms are those who have the intention and effect of helping students make sense of mathematics (Jaworski, 2004). Nevertheless, it must be taken into account that primary school teachers in Spain are not mathematicians, even if they have to teach mathematics. In order to become a primary school teacher in Spain, one must hold a bachelor's degree in primary education. In this degree, primary school teachers are trained in pedagogy, psychology, and specific didactics. Consequently, they need to receive proper basic instruction in mathematics and know the didactic aspects associated with the teaching-learning process.

In cases where primary education teachers without basic qualifications in mathematics are required to teach the subject, this may cause some learning difficulties among their students. Some publications have demonstrated that "teachers with limited subject knowledge have been shown to focus on a narrow conceptual field rather than on forging wider connections between the facts, concepts, structures and practices of mathematics" (Walshaw, 2012:182). It is over a decade now since Drake (2001) asserted that teaching mathematics requires that professionals in charge have a good level of knowledge of the field. Even today, teachers with a poor command of mathematics teach the subject in Spain, admittedly due (at least, partly) to the primary education teacher training curriculum. The curriculum is not governed by common guidelines and thus each university is autonomous regarding the development of their syllabus, resulting in great differences among universities (Jiménez, Ramos & Ávila, 2012; Rico Romero, Gómez Guzmán & Cañadas Santiago, 2014).

To teach school mathematics, mathematical knowledge is as necessary as pedagogical knowledge. Bromme (1994) indicates that mathematics teachers must possess knowledge about the following elements:

1) Mathematics as a field of study
2) School mathematics
3) Philosophy of school mathematics
4) General pedagogical (and psychological) knowledge
5) Pedagogical (or didactic) knowledge about the contents

Schoenfeld (2010:480) focuses expressly on the relevance of the latter point. "Knowing to anticipate specific student understandings and misunderstandings in specific instructional contexts, and having strategies ready to employ when students demonstrate those misunderstandings," is a fundamental aspect of quality teaching. For the pedagogical development of mathematics teachers, the examination of three fundamental factors is considered: available resources, orientations which they consider to be important, and goals which they are trying to achieve (Schoenfeld, Thomas & Barton, 2016).

In general terms, we can say that teacher trainees have a variable command of the concepts and procedures of the curriculum for primary school mathematics (knowledge about school mathematics) (Godino, Batanero, Roa & Wihelmi, 2008; Godino, Font, Wilhelmi & Lurduy, 2011). It is remarkable that such teachers, despite having the skills to functionally use basic arithmetic, struggle with conceptual issues like understanding that arithmetic is only the algorithms of basic operations or facing conceptual problems when using rational numbers and fractions (Flores, 2000). These kinds of issues result in university professors attending differently to the components of knowledge based on the students' command of mathematics (Flores, 1999).

Didactic knowledge about mathematical content provides the resources to synthesise in the area of actions, thought, theories, and principles over events in the classroom (Maz-Machado et al., 2015). Didactic knowledge of mathematics is essential for teaching them effectively and is also influenced by the strategies that teachers apply in the classroom (Ball & Bass, 2000).

Shulman (1986) notes that teachers' knowledge must be focused on the right command of the discipline as much as it is on the didactic orientation of its content. The didactic aspects should be addressed during initial teacher training and must be internalised by trainee teachers through practical experiences related to their future teaching practice (Walshaw, 2012).

Classroom experiences of that kind allow improvement not only on mathematical content but also on the interaction between teacher and student. However, which tasks, questions, and classroom activities train students to cope more easily with mathematical generalisation and abstraction? How

can those be recognised by teachers? (Jaworski, 1998).

Future teachers should be prepared to face various pedagogical and cognitive circumstances that their students will put forward in their future professional teaching. Nonetheless, it is evidently impossible to provide teachers at university level with all the potentially necessary knowledge to cope with all possible situations in all the facets of their future profession. Hence, it becomes a pressing need that students, while being guided by their teachers, are active participants in their own learning process, so that they may acquire and improve their pedagogic learning by themselves, thus developing diverse skills (Chamoso & Cáceres, 2009).

To achieve this, the presentation of routine activities does not suffice. On the contrary, it is necessary to carry out activities to develop mathematical capacity, team-work skills, communication of mathematical ideas, social skills, et cetera. It is important to take teamwork into account, and to raise awareness of the relevance of the communication and debate of ideas. The objective is to teach students to reason critically, solve complex problems and apply knowledge to real situations like those they might encounter in their teaching careers (Chamoso & Cáceres, 2009; Harkness, D'ambrosio & Morrone, 2007). Consequently, it is advisable to design practical activities that actively engage students to help them develop mathematical concepts that require reasoning and creativity, analysing information, discovering, and communicating ideas. Trainee teachers must thus develop mathematical teaching competences to become competent in teaching mathematics (Llinares, 2009).

From this point of view, becoming a successful mathematics teacher means to learn how to develop these competences, regarding their knowledge as much as their use (Llinares, 2004). These competences imply being aware of the kinds of answers to questions that are characteristic in learning mathematics, and to have an accurate idea of the expected answers (Niss & Højgaard, 2011).

Teacher trainees do frequently share the view that practical activities in a mathematics lesson are limited to the realisation of exercises or problems. However, they do not usually consider that such practice can often provide solutions to problematic situations from daily life. Moreover, taking on the role of problem-solvers helps students to identify the cognitive processes that are necessary to carry out a particular activity or to acquire a particular concept.

It follows that being able to influence and modify the beliefs of these trainee teachers can be essential for changing teaching practice in the classroom. Gaining deeper understanding of the nature of trainee teachers' beliefs about teaching and learning mathematics, as of the links between their beliefs and the practices they may bring into their classrooms, is vital to mathematics teacher training research (Buehl & Beck, 2015; Stipek, Givvin, Salmon & MacGyvers, 2001).

Along these lines, it must be taken into consideration that social and cultural characteristics have considerable influence on the perceptions of trainee teachers and that, hence, teacher training plans and programmes must be designed to respond to the created needs (Coultas & Lewin, 2002; Lin, Gorrell & Taylor, 2002; Locke, 2009).

Based on the views presented, the purpose of this study was to analyse the perceptions of trainee teachers towards practical mathematical lessons, after carrying out various practical activities in which manipulatives were used in the classroom, as part of their pedagogic and mathematic instruction. The objective of this study thus was to reveal teacher trainee's perceptions about the utility of mathematical practical lessons in which manipulatives are used.

As this research identified trainee teachers' perceptions about the utility of this kind of practical sessions, it contributes to teacher training by exposing the didactical aspects that trainee teachers struggle with, thus allowing developers of teacher training programmes to address these in designing mathematical practical sessions with manipulatives.

Methodology

Procedure

The research done was exploratory and descriptive; a qualitative methodology was used to analyse the perceptions of students towards mathematical practical sessions with manipulatives. The data was analysed using the content analysis methodology (Krippendorff, 1980).

During the 2015–16 academic year, as part of the modules of mathematics in the bachelor's degree in primary education, 12 practical mathematics lessons of 1 hour per week were presented. The teaching staff involved in these courses agreed to follow the same scheme for the work plan and the methodology for all groups of students.

For the practical mathematics lessons students were paired and received a handout or worksheet displaying an outline of the activities, indicating the topic, the objectives, and the necessary manipulatives (Cuisenaire rods, attribute blocks, geoboards, tangrams, fraction dominoes, physical materials to build space figures, polyhedral dice sets, etc.). At the same time, the manipulatives for use in the particular practice session were handed out to them. Students worked in pairs and only one worksheet per group was developed and submitted to the teacher at the end of the practical lesson.

The practical sessions were devised to promote team work and required the use of manipulatives. Students were required to make drawings or graph charts to illustrate their work with the materials. The professor in charge of the lesson, other

than clarifying and answering students' questions, took on the role of a guide. The time allotted to work on each practice was 1 hour after which the handouts and students' notes, results, considerations or conclusions were collected. The outlines, together with the evaluation of the practice, were then posted on the university's Moodle platform – to which student enrolled in the module had access – so that the students could review their work *post hoc*.

Population and Sample

The population comprised the students in the Primary Education Teaching Degree in the Faculty of Education at the University of Córdoba. The sample of the study comprised of 164 Year 1 students who studied the mathematics module during the 2015–16 academic year. Student participation in completing the questionnaire was voluntary and anonymous, so the sample was intentional and for convenience.

Instruments and Analysis

After completing the module, students were requested to write down their perceptions about the utility of participating in practical sessions using different manipulative resources and materials. The instrument used for collecting data was an online questionnaire with 10 open-ended items addressing several aspects regarding the use of manipulatives within the practical mathematics lessons. The questionnaire was administered individually during the last practical lesson at the end of the semester. In this paper we analyse one of the items regarding the perceived utility of the use of manipulatives in practical mathematics lessons. The item analysed was: *Indicate below the utility that the practical sessions carried out during the last year have*.

The information obtained was analysed using ATLAS.ti software (Version 7.5.4), which enabled us to establish categories of conceptual correlation.

To perform the analysis, the starting point was not any pre-established set of labels. On the contrary, labels emerged from the reading and analysis of the answers given by students and, later, they were consensually determined by experts in didactics of mathematics at the universities of Córdoba and Salamanca. In total, 23 labels or codes were assigned and grouped in categories according to their conceptual type. Figure 1 shows an example, in the original language of students' response, of how the answers led to the categorisation. These labels were later on translated into English for the purpose of presenting the results in this paper.

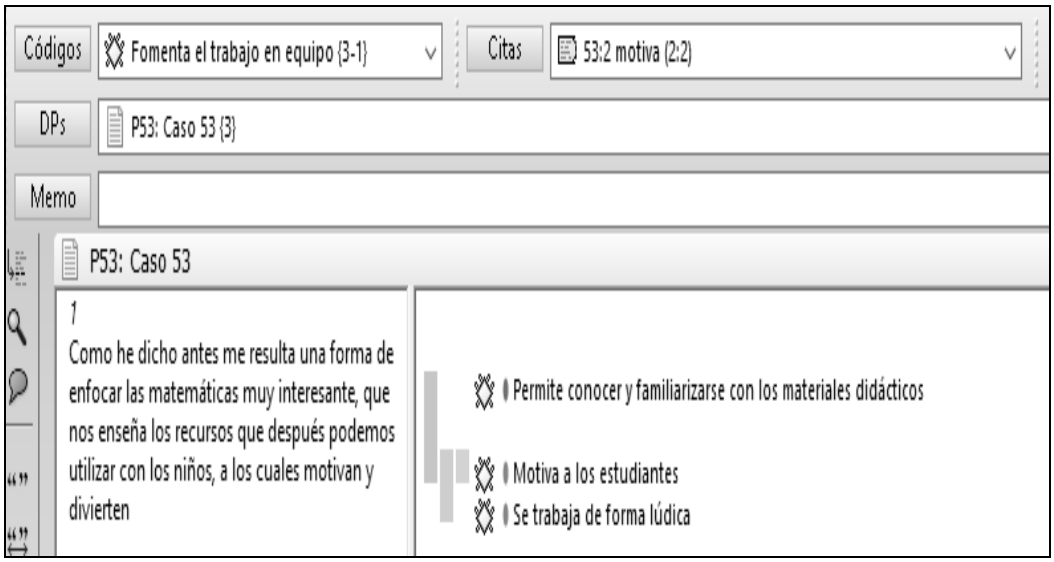

Figure 1 Codification of information in ATLAS.ti

This figure shows the labels assigned to Case P53, which reads "*As I said before, it seems to me a very interesting way to approach mathematics, which shows us the resources that we will later on be able to use with the children, whom they motivate and let have fun*" [*sic*].

Results

The answers received varied in several ways. Both succinct answers in a few words like "*They give us ideas for the future*" (Case 41), and long statements were received.

> *I think that the materials are useful to teach mathematics because they make the contents more familiar and tangible for the children, so that they can come to a better understanding of what mathematics are.*
>
> *The contents in mathematics are usually more abstract that in other subjects and thanks to the materials the children can understand them more easily and with lower levels of abstraction. I think that we*

> *must not encourage what has been done up to now, rote learning through repetition; we should support mathematics learning in such a way that it develops the cognitive capacity of the pupils, and this is enhanced by the materials we have used in class.* (Case 159)

Classifying the information into 23 labels resulted in six categories for grouping statements, which are described below.

Attitudinal Aspects

These are related to the predisposition and emotions expressed by the students regarding the practical sessions and in relation with how useful they are. Four types of labels are grouped under this category (Figure 2): *They help to forget your fear for mathematics; They avoid boredom; They motivate students; and They are a form of ludic work.* The latter is related with the previous two in the form of co-occurrence of labels.

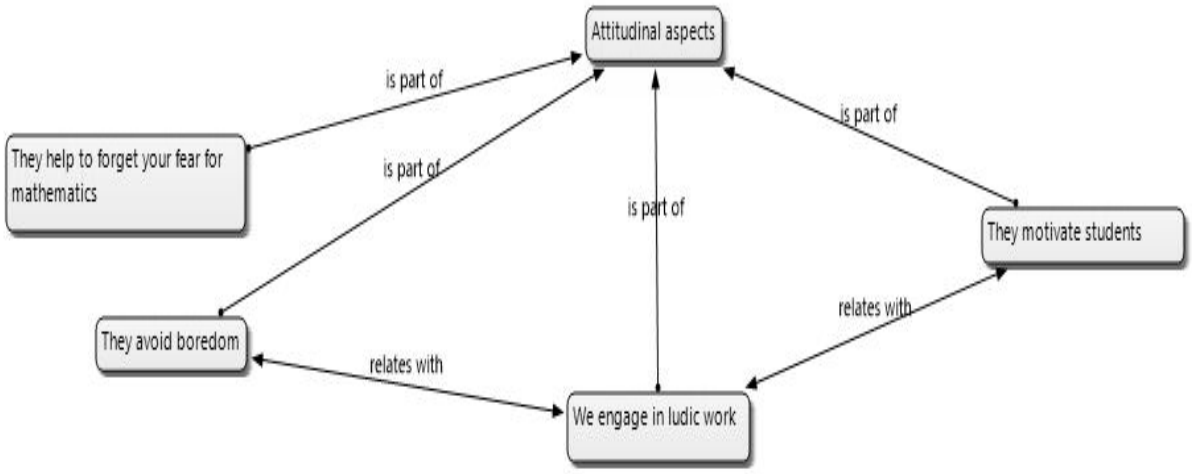

Figure 2 Utility of practical sessions regarding attitudinal aspects

For example, one of the students said that the utility of these practical sessions was: "[...] *they make you overcome your fear for them* [mathematics]" (Case 24).

Curricular Aspects

These refer to aspects that form part of the curriculum in mathematics but differ from methodological issues, and which the teacher must consider in planning the course. The three components found are the following: *They help to achieve the objectives of the subject; They make concepts visually perceptible; and They complement theory* (Figure 3). None of these labels were found with any of the others in any answers in this category; that is, they do not relate conjointly as useful in practical lessons regarding objectives and content. As an example, one student considered that with these sessions "*you can verify through experience what it is explained in the theory*" [*sic*] (Case 25).

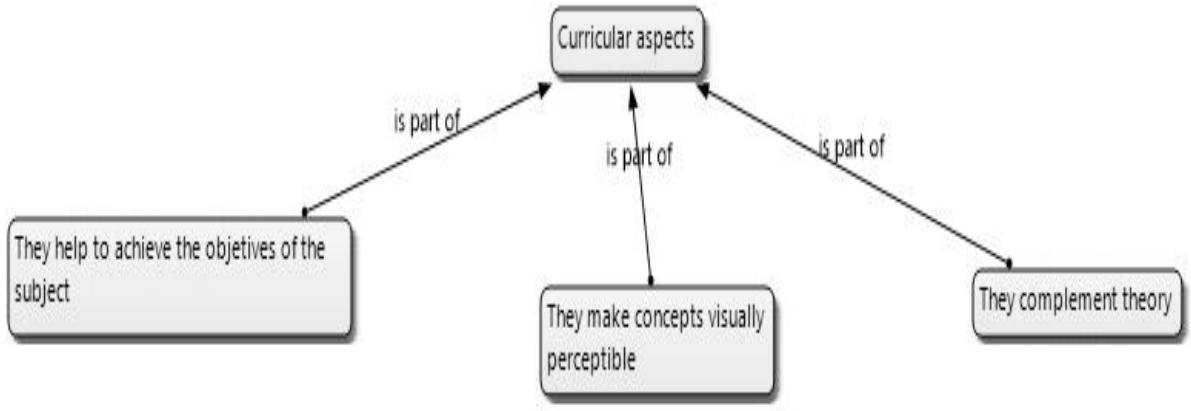

Figure 3 Utility of practical sessions regarding curricular aspects

Methodological Aspects

These aspects stress how practical sessions using manipulatives provide an overview of their potential impact on mathematics learning, and therefore highlight their importance to teaching, due to the variety of strategies that are put into play. This category groups the following labels: *Practice becomes easier; They pose an alternative to working with pen and paper; They facilitate experimentation;* and *They foster team working* [*sic*] (Figure 4). No co-occurrence of labels was found in any of the answers. For instance, one student answered: "*These practical sessions are also the key to foster team working* [*sic*] *and to promote relations between students*" (Case 116).

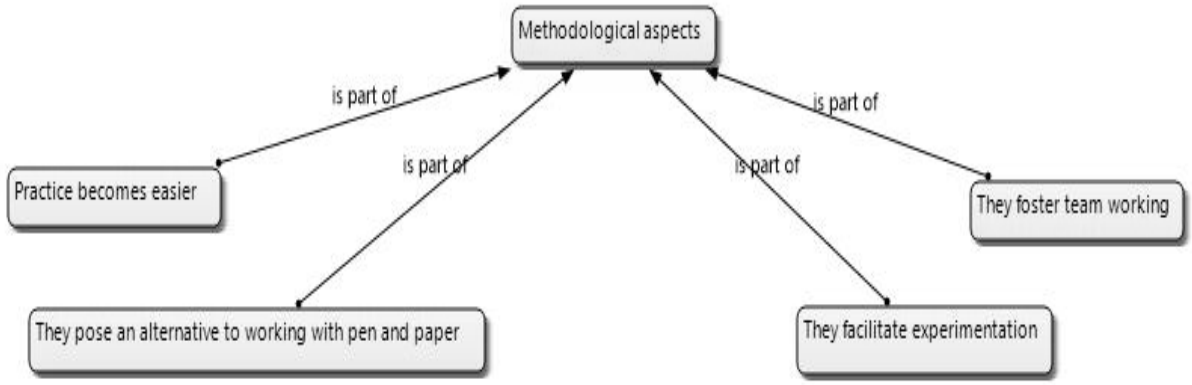

Figure 4 Utility of practical sessions regarding methodological aspects

Future Teaching Performance

This category groups all those arguments that are related to the teaching profession for which the students are training. Ideas about the knowledge of manipulatives and how students can transfer these experiences into their classroom once they become teachers, were identified. Five aspects (Figure 5) that relate to each other through co-occurrence of labels in the answers are presented in this category.

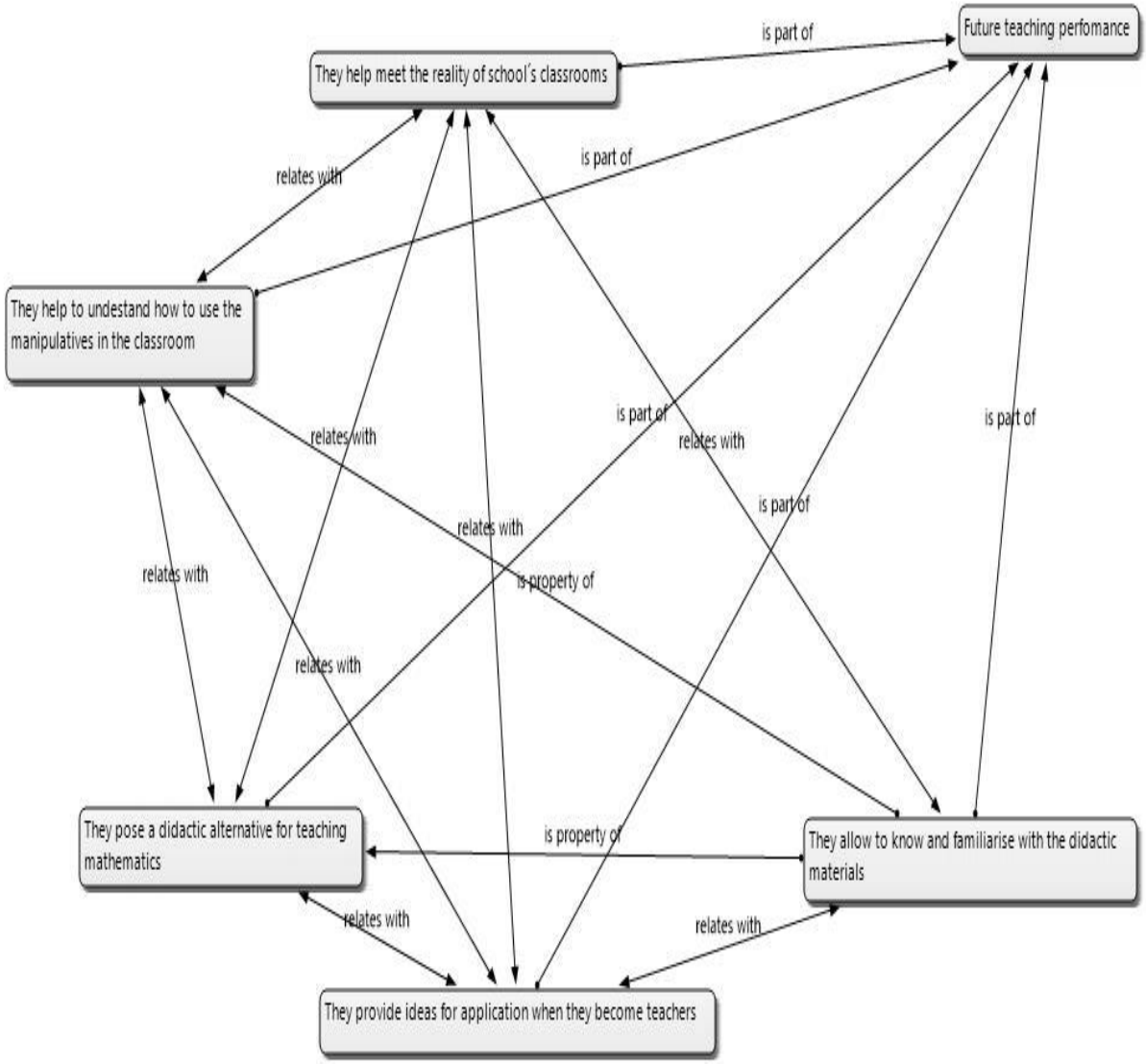

Figure 5 Utility of practical sessions regarding future teaching performance

For example, one student considered that these practical sessions "*are similar to the reality that we are going to find in the classrooms when we teach the subject*" (Case 30).

Learning Modes

These refer to how practical lessons make the learning of mathematics possible, either by making learning easier or more dynamic, understanding concepts, or facilitating meaningful learning (Figure 6). Co-occurrence was only found between the labels *Learning is easier* and *They facilitate comprehension of concepts*.

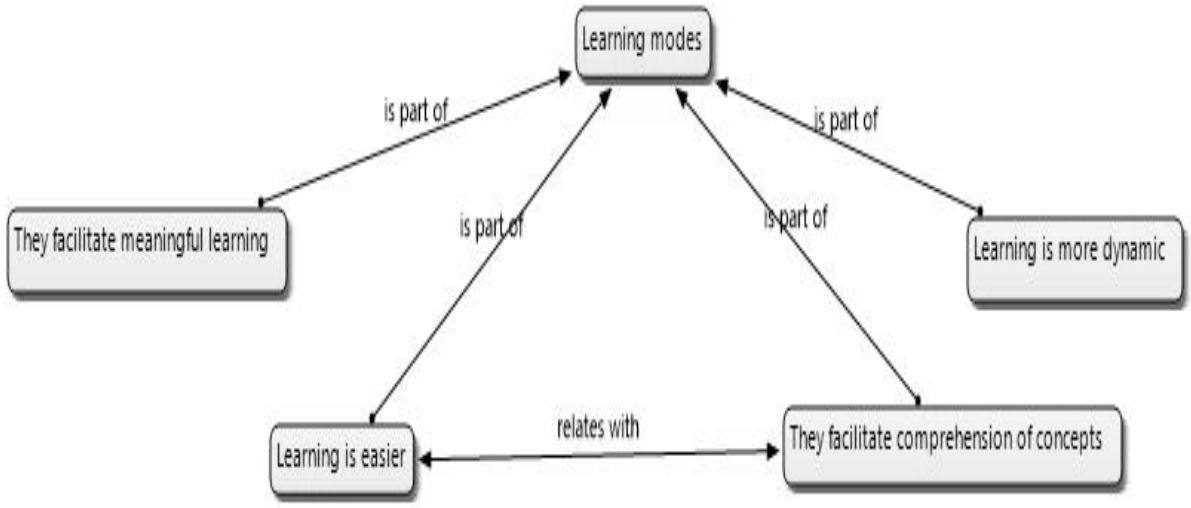

Figure 6 Utility of practical sessions regarding modes of learning

For example, a student stated that "*they foster a meaningful learning*" [*sic*] (Case 149).

Relation with Mathematics Itself

This category groups those ideas associated with labels in which students (to some extent) reflect about aspects of mathematics itself. It was thus revealed that the practical sessions in which they participated show the utility of mathematics, foster student engagement with mathematics, show that mathematics appears to be more familiar to the student and allow for hands-on manipulation of didactic materials and resources to visualise mathematics (Figure 7).

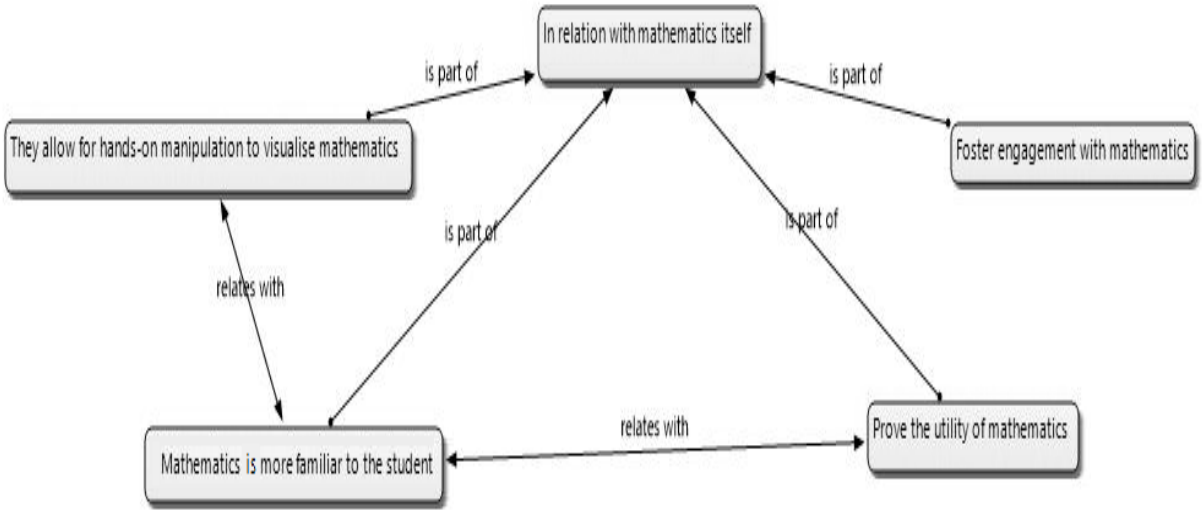

Figure 7 Utility of practical sessions regarding mathematics itself

As an example, a student stated: "*Children need to touch, see, hear ... They need to experience in order to build meaningful learning*" (Case 33) and these practical sessions foster that.

Based on the obtained answers, it was found that 56.1% of the trainee teachers marked *They facilitate comprehension of concepts* more than any of the possible other utilities. This utility presents co-occurrence with five other labels (Figure 8) relating to others from different categories like *relation with mathematics itself* or *curricular aspects*. The second most marked label was *They allow for manipulation to visualise mathematics* (27.4%), which related to 11 other labels (Figure 8).

Figure 8 Most repeated utility as pointed out by trainee teachers

Among the reasons why *They facilitate comprehension of concepts* was the most repeated answer is most likely the fact that Year 1 students focussed on the value of these practical sessions for their own instruction, disregarding (in some cases) their potential for future use.

Conclusion

The didactic knowledge that trainee teachers acquire in educational degrees must be oriented towards their acknowledgement of the importance of performing practices that help them master knowledge on the topics they are studying, that enriches the types of manipulatives that they will soon use in their professional lives, and that reinforces the tools and strategies that come into play in the discipline.

The students in the sample, despite being only in year 1 of their Primary Education Teaching Degree, already distinguish didactic and curricular aspects associated with teaching and learning mathematics. They highlight, as the principal utility of practical sessions, the easiness to learn from them, which provides evidence that they are aware of the difficulties and hindrances to the process of learning mathematics as well as that it is necessary to master the meaning of mathematical concepts in order to deal with them. Likewise, they value practical sessions for conveying information about alternative methodologies that strengthen their didactic knowledge on the matter and that they may present to their future pupils when they come across difficult and different situations in their teaching.

With regard to curricular aspects, the resolution of the practical tasks and the use of manipulatives constitute, for these students, a means to connecting theoretical mathematical knowledge to real problems in daily life. In engaging in these practices, students reflect about the utility of mathematics, which could in turn become a tool to improve the attitudes towards mathematics of education degrees.

As a complement to this work, further study could be aimed at determining what utility is assigned to the realisation of mathematical practical sessions by trainee teachers in upper courses (years 2, 3 …), and how greater general didactic knowledge and more specific didactics may influence student's considerations.

Authors' Contributions

AMM wrote the manuscript. CLM, NJF provided the data for the analysis. CLM and MJM conducted all qualitative analysis. AMM, NJF drew the conclusions. MJM and NJF edited the manuscript for proper English language. All authors reviewed the final manuscript.

Notes

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